JEE Advanced

Single Correct Answer Type

1. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is
   a. 1 **b.** 2 **c.** 3 **d.** none of these
   (IIT-JEE 1979)

2. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. The number of words which have at least one letter repeated is
   a. 59720 **b.** 79260 **c.** 69760 **d.** none of these
   (IIT-JEE 1982)

3. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to
   a. ${}^{47}C_5$ **b.** ${}^{52}C_5$ **c.** ${}^{52}C_4$ **d.** none of these
   (IIT-JEE 1982)

4. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First, the women choose the chairs from amongst the chairs marked 1 to 4, and then the men select the chairs from amongst the remaining. The number of possible arrangements is
   a. ${}^6C_3 \times {}^4C_2$ **b.** ${}^4P_2 \times {}^4P_3$ **c.** ${}^4C_2 + {}^4P_3$ **d.** none of these
   (IIT-JEE 1982)

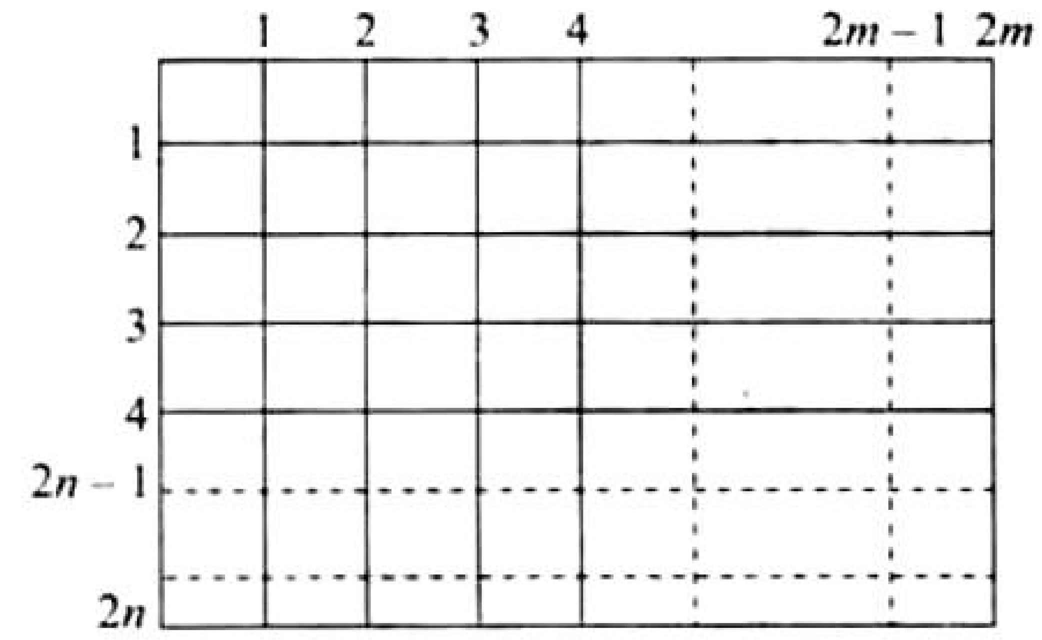
5. In a group of boys, two boys are brothers and six more boys are present in the group. In how many ways can they sit if the brothers are not to sit along with each other?
   a. $2 \times 6!$ **b.** ${}^7P_2 \times 6!$ **c.** ${}^7C_2 \times 6!$ **d.** none of these
   (IIT-JEE 1982)

6. A five-digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4, and 5, without repetition. The total number of ways this can be done is
   a. 216 **b.** 240 **c.** 600 **d.** 3125
   (IIT-JEE 1989)

7. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5, and 7. The smallest value of n for which this is possible is
   a. 6 **b.** 7 **c.** 8 **d.** 9
   (IIT-JEE 1998)

8. How many different nine-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?
   a. 16 **b.** 36 **c.** 60 **d.** 180
   (IIT-JEE 2000)

9 A rectangle with sides $2m - 1$ and $2n - 1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



a. $(m+n-1)^2$ b. 4^{m+n-1}
c. m^2n^2 d. $m(m+1)n(n+1)$

(IIT-JEE 2000)

10. Let T_n denote the number of triangles, which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals

a. 5 b. 7 c. 6 d. 4

(IIT-JEE 2001)

11. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is

a. 40 b. 60 c. 80 d. 100

(IIT-JEE 2002)

12. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is

a. 252 b. 254 c. 225 d. 224

(IIT-JEE 2006)

13. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

a. 360 b. 192 c. 96 d. 48

(IIT-JEE 2007)

14. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

a. 55 b. 66 c. 77 d. 88

(IIT-JEE 2009)

15. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to

a. 25 b. 34 c. 42 d. 41

(IIT-JEE 2010)

16. The total number of ways in which five balls of different colours can be distributed among three persons so that each person gets at least one ball is

a. 75 b. 150 c. 210 d. 243

(JEE Advanced 2012)

17. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

a. 264 b. 265 c. 53 d. 67

(JEE Advanced 2014)

18. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

a. 264 b. 265 c. 53 d. 67

(JEE Advanced 2014)

Linked Comprehension Type

For Problems 1–2

Let n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n= the number of such n-digit integers ending with digit 1 and c_n= the number of such n-digit integers ending with digit 0. (IIT-JEE 2012)

1. The value of b_6 is

a. 7 b. 8 c. 9 d. 11

2. Which of the following is correct?

a. $a_{17} = a_{16} + a_{15}$ b. $c_{17} \neq c_{16} + c_{15}$
c. $b_{17} \neq b_{16} + c_{16}$ d. $a_{17} = c_{17} + b_{16}$

Matching Column Type

1. Consider all possible permutations of the letters of the word ENDEANOEL.

Column I	Column II
(a) The number of permutations containing the word ENDEA is	**(p)** 5!
(b) The number of permutations in which the letter E occurs in the first and the last positions is	**(q)** $2 \times 5!$
(c) The number of permutations in which none of the letters D, L, N occurs in the last five positions is	**(r)** $7 \times 5!$
(d) The number of permutations in which the letters A, E, O occur only in odd positions is	**(s)** $21 \times 5!$

(IIT-JEE 2008)

Integer Answer Type

1. Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is

(JEE Advanced 2013)

2. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Color the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

(JEE Advanced 2014)

3. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

(JEE Advanced 2014)

4. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

(JEE Advanced 2015)

Fill in the Blanks Type

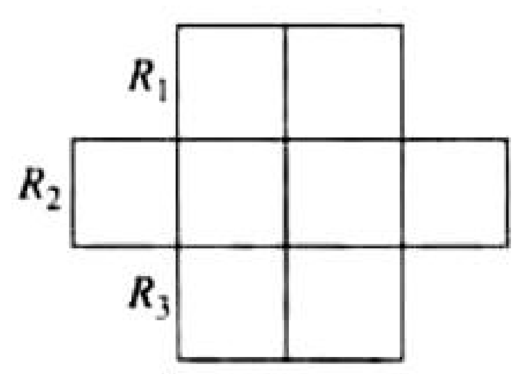
1. In a certain test, a_i students gave wrong answers to at least i questions, where $i = 1, 2, ..., k$. No student gave more than k wrong answers. The total number of wrong answers given is ——. **(IIT-JEE 1982)**
2. The sides AB, BC, and CA of a triangle ABC have 3, 4, and 5 interior points, respectively, on them. The number of triangles that can be constructed using these interior points as vertices is ——. **(IIT-JEE 1984)**
3. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is ——. **(IIT-JEE 1988)**
4. There are four balls of different colors and four boxes of colors same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own color is ——. **(IIT-JEE 1992)**

True/False Type

1. The product of any r consecutive natural numbers is always divisible by $r!$. **(IIT-JEE 1985)**

Subjective Type

1. **(i)** In how many ways can a pack of 52 cards be divided equally among four players?
   (ii) In how many ways can you divide these cards in four sets, three of them having 17 cards each and the fourth one just one card? **(IIT-JEE 1979)**
2. Six Xs have to be placed in the squares of figure adjacent in such a way that each row contains at least one X. In how many different ways can this be done?



(IIT-JEE 1978)

3. Five balls of different colors are to be placed in the boxes of different size. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty? **(IIT-JEE 1981)**
4. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is $m!(m + 1)!/(m - n + 1)!$. **(IIT-JEE 1983)**
5. Seven relatives of a man comprises four ladies and three gentlemen; his wife has also seven relatives—three of them are ladies and four gentlemen. In how many ways can they invite 3 ladies and 3 gentlemen at a dinner party so that there are three man's relatives and three wife's relatives? **(IIT-JEE 1985)**
6. A box contains two white balls, three black balls, and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw? **(IIT-JEE 1986)**
7. A student is allowed to select almost n books from a collection of $2n + 1$ books. If the total number of ways in which he can select at least one book is 63, find the value of n. **(IIT-JEE 1987)**
8. A number of 18 guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the sitting arrangements can be made. **(IIT-JEE 1991)**
9. A committee of 12 is to be formed from nine women and eight men. In how many ways can this be done if at least five women have to be included in a committee? In how many of these committees
   a. the women hold majority?
   b. the men hold majority? **(IIT-JEE 1994)**
10. Prove by permutation or otherwise that $(n^2)!/(n!)^n$ is an integer $(n \in I^+)$. **(IIT-JEE 2004)**

Answer Key

JEE Advanced

Single Correct Answer Type

1. c.	**2.** c.	**3.** c.	**4.** d.	**5.** b.
6. a.	**7.** b.	**8.** c.	**9.** c.	**10.** b.
11. a.	**12.** c.	**13.** c.	**14.** c.	**15.** d.
16. b.	**17.** c.	**18.** c.		

Linked Comprehension Type

1. b. **2.** a.

Matching Column Type

1. (a) – (p); (b) – (s); (c) – (q); (d) – (q)

Integer Answer Type

1. (5) **2.** (5) **3.** (7) **4.** (5)

Fill in the Blanks Type

1. $a_1 + a_2 + a_3 + \ldots + a_k$ **2.** 205
3. 35 **4.** 9

True/False Type

1. True

Subjective Type

1. i. $\dfrac{52!}{(13!)^4}$ **ii.** $\dfrac{52!}{(17!)^3 1!3!}$

2. 26 **3.** 150 **4.** $\dfrac{(m+1)!m!}{(m-n+1)!}$

5. 485 **6.** 64 **7.** 3

8. ${}^{11}C_5 \times 9! \times 9!$ **9.** 6062 **a.** 2702 **b.** 1008

JEE Advanced

Single Correct Answer Type

1. c. ${}^nC_{r-1} = 36, {}^nC_r = 84, {}^nC_{r+1} = 126$

We know that

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{r}{n-r+1}$$

$$\therefore \quad \frac{36}{84} = \frac{r}{n-r+1}$$

or $\quad \dfrac{r}{n-r+1} = \dfrac{3}{7}$

or $\quad 3n - 10r + 3 = 0 \qquad (1)$

Also,

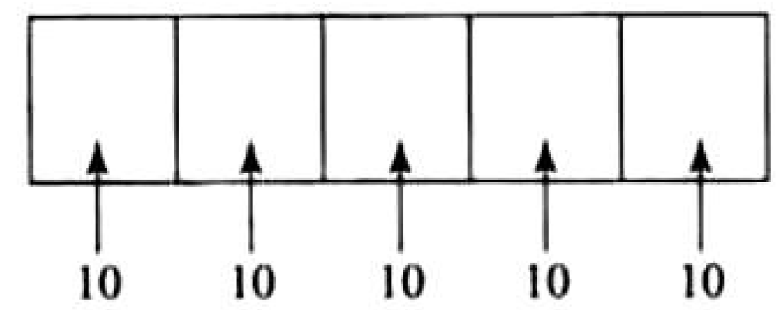
$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r} = \frac{84}{126} = \frac{2}{3}$$

or $\quad 2n - 5r - 3 = 0 \qquad (2)$

Solving (1) and (2), we get $n = 9$ and $r = 3$.

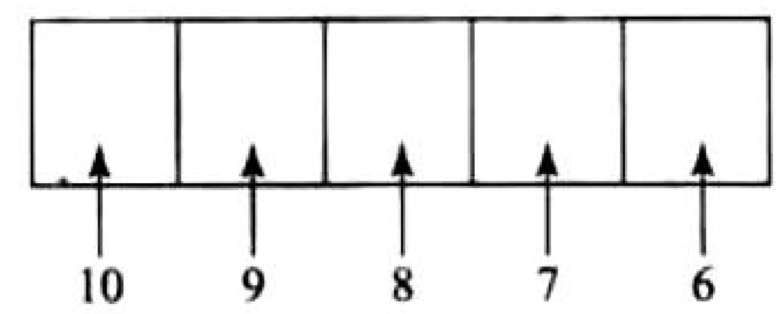
2. c. Number of words when repetition is allowed is

$10 \times 10 \times 10 \times 10 \times 10 = 10^5$.



Number of words when repetition is not allowed is

$10 \times 9 \times 8 \times 7 \times 6 = 30240$.



Hence, required number of words in which at least one letter is repeated is $100000 - 30240 = 69760$.

3. c. $${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4)$

[Using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ({}^{48}C_3 + {}^{48}C_4)$

$= {}^{51}C_3 + {}^{50}C_3 + ({}^{49}C_3 + {}^{49}C_4)$

$= {}^{51}C_3 + ({}^{50}C_3 + {}^{50}C_4)$

$= {}^{51}C_3 + {}^{51}C_4$

$= {}^{52}C_4$

4. d. $\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}\,\bar{5}\,\bar{6}\,\bar{7}\,\bar{8}$

Two women can choose two chairs out of 1, 2, 3, 4 in ${}^4C_2$ ways, and can arrange among themselves in 2! ways. Three men can choose 3 chairs out of 6 remaining chairs in ${}^6C_3$ ways and can arrange themselves in 3! ways.

Therefore, total number of possible arrangements is

${}^4C_2 \times 2! \times {}^6C_3 \times 3! = {}^4P_2 \times {}^6P_3$.

5. b. $\times B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times B_6 \times$

Let first six boys sit, which can be done in 6! ways. Once they have been seated, the two brothers can be made to occupy seats in between or in extreme (i.e. on crosses) in ${}^7P_2$ ways.

Hence, required number of ways is ${}^7P_2 \times 6!$.

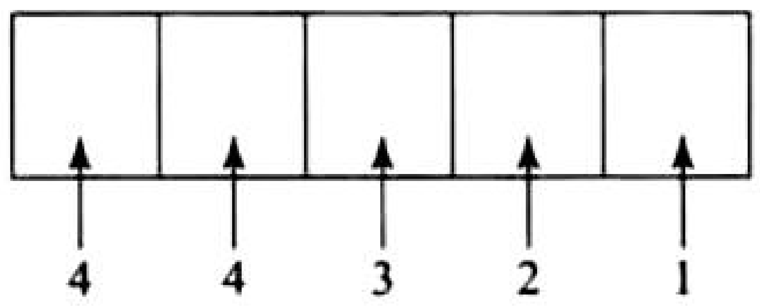
6. a. We know that a number is divisible by 3 if the sum of its digits is divisible by 3. Now out of 0, 1, 2, 3, 4, 5 if we take 1, 2, 3, 4, 5 or 0, 1, 2, 4, 5, then the 5-digit numbers will be divisible by 3.

Case I:

Number of five-digit numbers formed using the digits 1, 2, 3, 4, 5 is 5! = 120.

Case II:

Taking 0, 1, 2, 4, 5, number of five digit numbers is $4 \times 4! = 96$.



From case I and case II, total number divisible by 3 is $120 + 96 = 216$.

7. b. Distinct n-digit numbers which can be formed using digits 2, 5, and 8 are 3^n. We have to find n so that

$3^n \geq 900$

or $3^{n-2} \geq 100$

or $n - 2 \geq 5$

or $n \geq 7$

So the least value of n is 7.

8. c. X – X – X – X – X

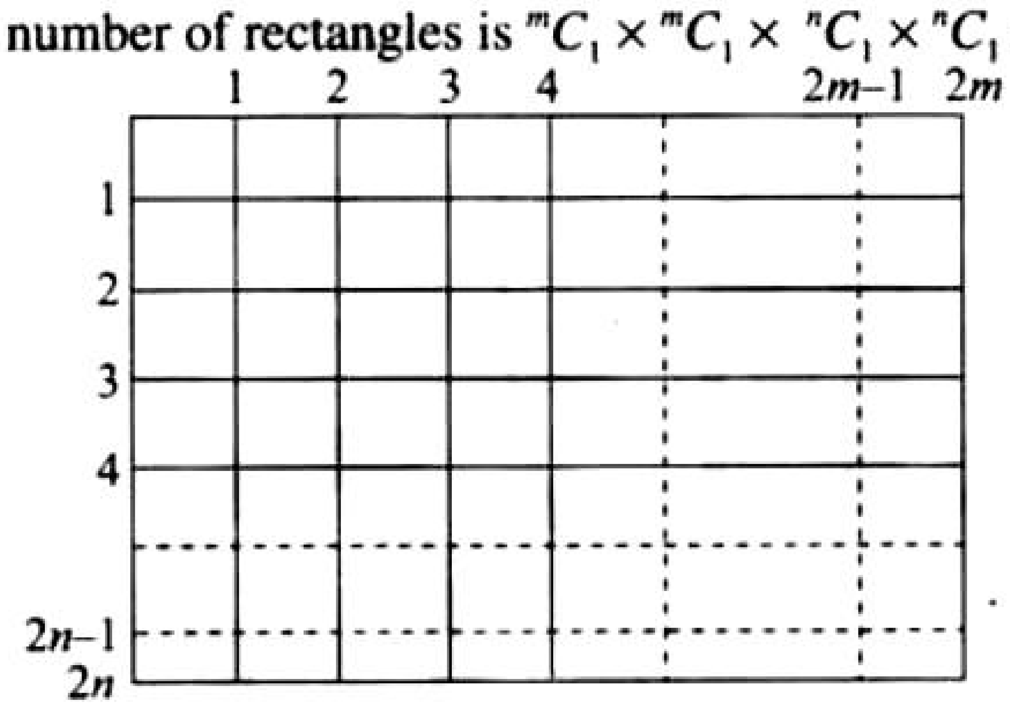
The four digits 3, 3, 5, 5 can be arranged at (–) places in $\frac{4!}{2!2!} = 6$ ways. The five digits 2, 2, 8, 8, 8 can be arranged at (X) place in $\frac{5!}{2!3!} = 10$ ways.

Total number of arrangements is $6 \times 10 = 60$.

9. c. If we see the blocks in terms of lines, then there are $2m$ vertical lines and $2n$ horizontal lines.

To form the required rectangle with odd side length we must select two horizontal lines, one even numbered (out of 2, 4, ..., $2n$) and one odd numbered (out of 1, 3, ..., $2n - 1$) and similarly two vertical lines.

The number of rectangles is ${}^mC_1 \times {}^mC_1 \times {}^nC_1 \times {}^nC_1 = m^2n^2$



10. b. A regular polygon of n sides has n vertices, no two of which are collinear. Out of these n points, ${}^nC_3$ triangles can be formed.

$\therefore \quad T_n = {}^nC_3;\ T_{n+1} = {}^{n+1}C_3$

Given,

$T_{n+1} - T_n = 21$

or ${}^{n+1}C_3 - {}^nC_3 = 21$

or $\frac{(n+1)n(n-1)}{3 \times 2 \times 1} - \frac{n(n-1)(n-2)}{3 \times 2 \times 1} = 21$

or $n(n-1)(n+1-n+2) = 126$

or $n(n-1) = 42$

or $n(n-1) = 7 \times 6$

or $n = 7$

11. a. Total number of ways of arranging the letters of the word BANANA is $\frac{6!}{2!3!} = 60$.

Number of words in which 2N's come together [(NN), B, A, A, A] is $5!/3! = 20$.

Hence, the required number is $60 - 20 = 40$.

12. c. If L.C.M. of p and q is $r^2t^4s^2$, then distribution of factors r is as follows:

p	q
r^0	r^2
r^1	r^2
r^2	r^2
r^2	r^0
r^2	r^1

Thus, factor r can be distributed in $2 \times 3 - 1$ ways. Similarly, factors t and s can be distributed in $2 \times 5 - 1$ and $2 \times 3 - 1$ ways, respectively.

Hence, number of ordered pairs are $(2 \times 3 - 1) \times (2 \times 5 - 1) \times (2 \times 3 - 1) = 225$.

13. c. The letters of word COCHIN in alphabetic order are C, C, H, I, N, O. Fixing first letter C and keeping C at the second place, rest 4 can be arranged in 4! ways.

Similarly, the total number of words starting with CH, CI, CN is 4! in each case.

Then fixing first two letters as CO, next four places when filled in alphabetic order gives the word COCHIN.

Therefore, number of words coming before COCHIN is

$4 \times 4! = 4 \times 24 = 96.$

14. c. The digits are 1, 1, 1, 1, 1, 2, 3

or 1, 1, 1, 1, 2, 2, 2

Hence number of seven digits numbers formed

$$= \frac{7!}{5!} + \frac{7!}{4!\,3!} = 77.$$

Alternative Method:

Required number of numbers

$= \text{coefficient of } x^{10} \text{ in } (x + x^2 + x^3)^7$

$= \text{coefficient of } x^3 \text{ in } (1 + x + x^2)^7$

$= \text{coefficient of } x^3 \text{ in } (1 - x^3)^7(1 - x)^{-7}$

$= {}^{7+3-1}C_3 - 7$

$= {}^9C_3 - 7 = \frac{9 \times 8 \times 7}{6} - 7 = 77$

15. d. $S = \{1, 2, 3, 4\}$

Each element can be put in 3 ways either in subsets or we don't put in any subset.

So total number of unordered pairs $= \frac{3 \times 3 \times 3 \times 3 - 1}{2} + 1 = 41$ ways (both subsets can be empty also)

16. b. Distribution of five balls among three persons is to be done in such a way that each gets at least one ball.

So number of balls received by persons is either 1, 2, 2 or 1, 1, 3.

$$\text{So, number of ways} = \left(\frac{5!}{1! \times (2!)^2 \times 2!} + \frac{5!}{3! \times (1!)^2 \times 2!}\right)3!$$

$$= 150$$

17. c. If '2' goes in '1' then it is derangement of 4 things which can be done in $4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9$ ways

If '2' doesn't go in '1', it is derangement of 5 things which can be done in $5!\left(\frac{1}{2!} - \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!}\right) = 44$ ways.

Hence, total 53 ways are there.

18. c. If '2' goes in '1' then it is derangement of 4 things which can be done in $4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)=9$ ways.

If '2' doesn't go in '1', it is derangement of 5 things which can be done in 44 ways.

Hence, total 53 ways are there.

Linked Comprehension Type

1.(b), 2.(a)

a_n= number of all n digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are zero.

This means that the number will end with 0 or 1.

Case I:

If the number ends with 0, then

$(n-1)^{th}$ digit should be 1

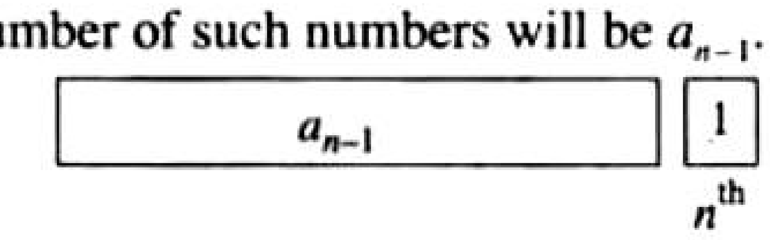
Hence number of such numbers will be b_{n-1}.

Case II:

If the number ends with 1, then

First $(n-1)$ digits should be $(n-1)$ digit positive integers formed by the digits 0, 1 or both such that no consecutive digits among them are zero.

Hence number of such numbers will be a_{n-1}.



$\therefore \quad a_n = a_{n-1} + b_{n-1}$ (1)

Since b_n is the number of such numbers which will end with 1, first $(n-1)$ digits will be $(n-1)$ digit positive integers formed by the digits 0, 1 or both such that no consecutive digits among them are zero.

Hence $b_n = a_{n-1}$

From(1),we get

$a_n = a_{n-1} + a_{n-2}$ and for $n = 17$, $a_{17} = a_{16} + a_{15}$

c_n is the number of numbers ending with 0.

So, $(n-1)^{th}$ digit should be 1.

First $(n-1)$ digits will be $(n-1)$ digit positive integers formed by the digits 0, 1 or both such that no consecutive digits among them are zero which will end with 1.

$\therefore \quad c_n = b_{n-1}$

$b_6 = a_5 = a_4 + a_3 = 2a_3 + a_2 = 3a_2 + 2a_1$

$a_2 = 2$ (10 and 11)

$a_1 = 1$ (1)

$\therefore \quad b_6 = 8$

Matching Column Type

1. (a) – (p); (b) – (s); (c) – (q); (d) – (q).

(a) ENDEA, N, O, E, L are five different letter which can be permuted in 5! ways.

(b) If E is in the first and last position then remaining letters N, D, A, N, O, E, L can be arranged in $\frac{7!}{2!}=21\times 5!$.

(c) We have (E, E, E), (N, N), D, A, O, L.

For first four positions, we have letters, D, L, N, N which can be arranged in $\frac{4!}{2!}$ ways.

For last five positions, we have letters, E, E, E, A, O which can be arranged in $\frac{5!}{3!}$.

So, total number ways $=\frac{4!}{2!}\times\frac{5!}{3!}=2\times 5!$

(d) Letters A, E, E, E, O can be arranged in five odd places in $\frac{5!}{3!}$ ways.

And remaining four letters, N, N, D, L can be arranged in $\frac{4!}{2!}$ ways

Hence, total number of ways $=\frac{5!}{3!}\times\frac{4!}{2!}=2\times 5!$

Integer Answer Type

1.(5) 8 vectors of given type are as follows:

$(1, 1, 1) \leftrightarrow (-1, -1, -1)$

$(1, 1, -1) \leftrightarrow (-1, -1, 1)$

$(1, -1, 1) \leftrightarrow (-1, 1, -1)$

$(-1, 1, 1) \leftrightarrow (1, -1, -1)$

The given pairs are collinear (anti parallel) any three pairs can be selected from the four available pairs and from each pair any one vector can be selected.

$\therefore \quad {}^4C_3 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 32 = 2^5 = 2^p$(given)

$\therefore \quad p = 5$

2. (5) Number of red lines $= {}^nC_2 - n$

Number of blue lines $= n$

Hence, ${}^nC_2 - n = n$

$\Rightarrow \quad {}^nC_2 = 2n$

$\Rightarrow \quad \frac{n(n-1)}{2} = 2n$

$\Rightarrow \quad n - 1 = 4 \Rightarrow n = 5$

3. (7) Possible solutions are

1, 2, 3, 4, 10

1, 2, 3, 5, 9

1, 2, 3, 6, 8

1, 2, 4, 5, 8

1, 2, 4, 6, 7

1, 3, 4, 5, 7

2, 3, 4, 5, 6

Hence, 7 solutions are there.

4. (5) For n

$B_1, B_2, B_3, B_4, B_5, (G_1, G_2, G_3, G_4, G_5)$

Number of arrangements $= n = 5! \times 6!$

For m

First arrange 5 boys in 5! ways.

$\uparrow B_1 \uparrow B_2 \uparrow B_3 \uparrow B_4 \uparrow B_5 \uparrow$

Now, we have to arrange 5 girls in such a way that group of four girls and the fifth girl are arranged in any two of the six positions shown as arrows.

Two positions can be selected in ${}^6C_2$ ways.

Four girls can be selected in ${}^5C_4$ ways.

Now, this group and the fifth girl can be arranged in selected two positions is 2! ways.

Also, four girls arrange among themselves in 4! ways.

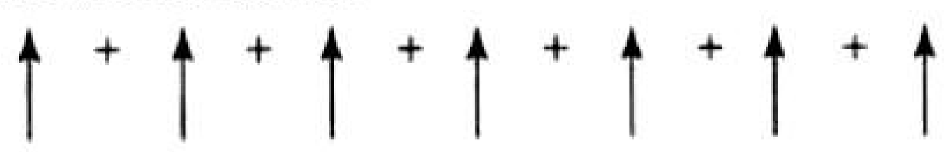
Hence, number of arrangements $= m = 5! \times {}^6C_2 \times {}^5C_4 \times 2! \times 4!$

$= 5! \times 15 \times 2 \times 5!$

$\therefore \quad \frac{m}{n} = \frac{5! \times 15 \times 2 \times 5!}{5! \times 6!} = 5$

Fill in the Blanks Type

1. Number of students who gave wrong answers to exactly one question is $a_1 - a_2$.
   Number of students who gave wrong answers to exactly two questions is $a_2 - a_3$.
   Number of students who gave wrong answers to exactly three questions is $a_3 - a_4$.
   Number of students who gave wrong answers to exactly k question is $a_{k-1} - a_k$.
   Therefore, total number of wrong answers is
   $$1(a_1 - a_2) + 2(a_2 - a_3) + 3(a_3 - a_4) + \cdots + k(a_{k-1} - a_k)$$
   $$= a_1 + a_2 + a_3 + \cdots + a_k$$
2. We have total $3 + 4 + 5 = 12$ points. So, number of Δs that can be formed using 12 such points is given by
   Total number of ways of selecting three point – Number of ways three collinear points are selected
   $$= {}^{12}C_3 - {}^{3}C_3 - {}^{4}C_3 - {}^{5}C_3$$
   $$= \frac{12\times11\times10}{6} - 1 - 4 - \frac{5\times4}{2\times1}$$
   $$= 220 - 15 = 205$$
3. Six '+' signs can be put in a row in one way creating seven gaps shown as arrows:



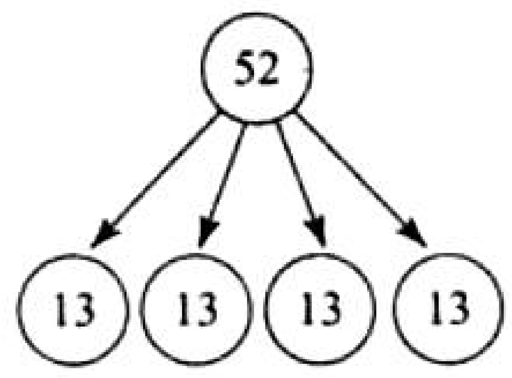
   Now 4 '–' signs must be kept in these gaps, so, no two '–' signs should be together.
   Out of these 7 gaps 4 can be chosen in ${}^7C_4$ ways. Hence, required number of arrangements is
   $${}^7C_4 = {}^7C_3 = \frac{7\times6\times5}{3\times2\times1} = 35$$
4. We know that number of derangements of n objects is
   $$n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + (-1)^n\frac{1}{n!}\right]$$
   Therefore, number of ways of putting all the 4 balls into boxes of different colour is
   $$4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 4!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$$
   $$= 24\left(\frac{12-4+1}{24}\right)$$
   $$= 9$$

True/False Type

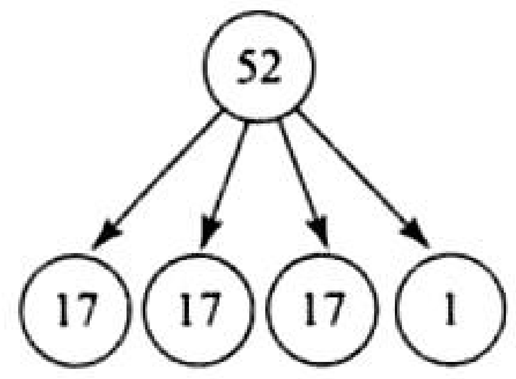
1. **True**
   Let the r consecutive integers be $m, m+1, m+2, \ldots, m+r-1$.
   We have $m(m+1)(m+2)\ldots(m+r-1)$
   $$= \frac{(m-1)!\, m(m+1)\cdots(m+r-1)}{(m-1)!}$$
   $$= \frac{(m+r-1)!}{(m-1)!}$$
   $$= r!\,\frac{(m+r-1)!}{(m-1)!\,k!}$$
   $$= (r!)^{(m+r-1)}C_r$$
   since ${}^{(m+r-1)}C_r$ is an integer, if follows that $r!$ divides $m(m+1)\ldots(m+r-1)$.

Subjective Type

1. **(i)** Distribution of 52 cards can be equally divided among four players.
   Hence, number of ways is
   $$\frac{52!}{(13!)4!}4! = \frac{52!}{(13!)}.$$

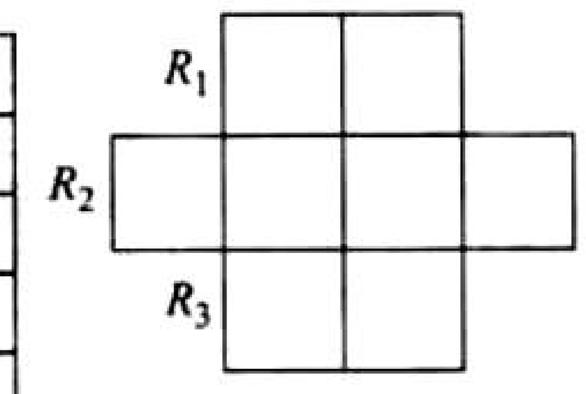


   (ii) Number of divisions is
   $$\frac{52!}{(17!)^3\,1!3!}.$$

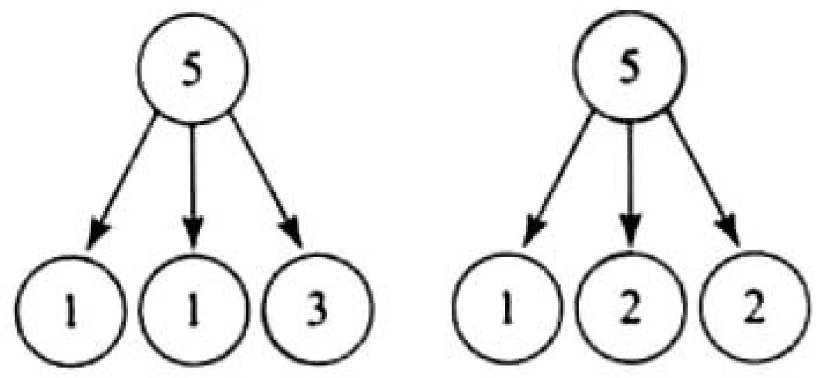


   Note: There is division by 3! since 3 groups can be arranged in 3! ways and here 3 groups are of equal number of cards.
2. As all the X's are identical, the question is of selection of 6 squares from 8 squares, so that no row remains empty. Here R_1 has 2 squares, R_2 has 4 squares, and R_3 has 2 squares. The selection scheme for squares is as follows:

R_1	R_2	R_3
1	4	1
1	3	2
2	3	1
2	2	2



   Therefore, number of selection is
   $${}^2C_1\times{}^4C_4\times{}^2C_1 + {}^2C_1\times{}^4C_3\times{}^2C_2 + {}^2C_2\times{}^4C_3\times{}^2C_1 + {}^2C_2\times{}^4C_2\times{}^2C_2 = 4 + 8 + 8 + 6 = 26$$
3. As no box should remain empty, boxes can have balls in the following numbers:
   Possibilities 1, 2, 3 or 1, 2, 2



   Division ways for tree (i) is $\frac{5!}{(1!)^2\,3!2!}$
   Division ways for tree (ii) is $\frac{5!}{(2!)^2\,1!2!}$
   Now, total number of ways of distribution of these groups into three boxes is
   $$\left[\frac{5!}{(1!)^2\,3!2!} + \frac{5!}{(2!)^2\,1!2!}\right]\times 3! = 150$$
4. m men can be seated in $m!$ ways, creating $(m+1)$ for ladies. Now n ladies can be seated in $(m+1)$ places (as $n < m$) in ${}^{m+1}P_n$ ways.
   Therefore, total number of ways is
   $$m!\times{}^{m+1}P_n = m!\times\frac{(m+1)!}{(m+1-n)!} = \frac{(m+1)!\,m!}{(m-n+1)!}$$

5. The scheme is as follows:

Husband's relatives		Wife's relatives		Number of selections
Male (3)	Female (4)	Male (4)	Female (3)	
3	0	0	3	${}^3C_3 \times {}^3C_3 = 1$
2	1	1	2	${}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 = 144$
1	2	2	1	${}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 = 324$
0	3	3	0	${}^4C_3 \times {}^4C_3 = 16$
			Total	485

6. Out of 2 white, 3 black, and 4 red balls, three balls have to be drawn.

If at least one black ball is selected, then we have following cases:

Black balls (3)	White + red balls (6)	Number of ways of selection
1	2	${}^3C_1 \times {}^6C_2 = 45$
2	1	${}^3C_2 \times {}^6C_1 = 18$
3	0	${}^3C_3 \times {}^6C_0 = 1$
	Total	64

7. Number of ways in which a student can select at least one and almost n books out of $2n + 1$ books is

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \cdots + {}^{2n+1}C_n$$

$$= \frac{1}{2}[2 \times {}^{2n+1}C_1 + 2 \times {}^{2n+1}C_2 + 2 \times {}^{2n+1}C_3 + \cdots + 2 \times {}^{2n+1}C_n]$$

$$= \frac{1}{2}\Big[({}^{2n+1}C_1 + {}^{2n+1}C_{2n}) + ({}^{2n+1}C_2 + {}^{2n+1}C_{2n-1}) + ({}^{2n+1}C_3 + {}^{2n+1}C_{2n-2}) + \cdots + ({}^{2n+1}C_n + {}^{2n+1}C_{n+1})\Big]$$

[Using ${}^nC_r = {}^nC_{n-r}$]

$$= \frac{1}{2}\Big[{}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \cdots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \cdots + {}^{2n+1}C_{2n}\Big]$$

$$= \frac{1}{2}\Big[{}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \cdots + {}^{2n+1}C_{2n+1} - 1 - 1\Big]$$

$$= \frac{1}{2}\big[2^{2n+1} - 2\big] = 2^{2n} - 1$$

Now given,

$2^{2n} - 1 = 63$

or $2^{2n} = 64 = 2^6$

or $2n = 6$

or $n = 3$

8. Out of 18 guests, 9 are to be seated on side A and rest 9 on side B.

Now out of 18 guests, 4 particular guests desire to sit on one particular side, say side A, and other 3 on other side B. Out of rest $18 - 4 - 3 = 11$ guests, we have to select 5 more for side A and rest 6 can be seated on side B. Selection of 5 out of 11 can be done in ${}^{11}C_5$ ways. Nine guests on each side of table can be seated in $9! \times 9!$ ways.

Thus, there are total ${}^{11}C_5 \times 9! \times 9!$ arrangements.

9. A committee of 12 is to be formed from 9 women and 8 men with minimum 5 women. Then we have following selection ways.

Women (9)	Men (8)	Number of ways of selecting
5	7	${}^9C_5 \times {}^8C_7 = 1008 = s_1$
6	6	${}^9C_6 \times {}^8C_6 = 2352 = s_2$
7	5	${}^9C_7 \times {}^8C_5 = 2016 = s_3$
8	4	${}^9C_8 \times {}^8C_4 = 630 = s_4$
9	3	${}^9C_9 \times {}^8C_3 = 56 = s_5$
	Total	6062

a. Number of committee when women are in majority is $s_3 + s_4 + s_5 = 2702$.

b. Number of committee when men are in majority is $s_1 = 1008$.

10. Let there be n sets of different objects, each set containing n identical objects, e.g., ((1, 1, 1, ..., 1(n times)), (2, 2, 2, ..., 2 (n times)), ..., (n, n, n, ..., n (n times)). Then the number of ways in which these $n \times n = n^2$ objects can be arranged in a row is

$$\frac{(n^2)!}{n!n!\cdots n!} = \frac{(n^2)!}{(n!)^n}$$

But, these number of ways should be a natural number. Hence, $(n^2)!/(n!)^n$ is an integer ($n \in I^+$).

Alternative Method 1:

Consider n^2 distinct object to be distributed among n persons if each gets n number of objects.

Then number of ways of distribution $= \left[\dfrac{(n^2)!}{(n!)^n n!}\right] n! = \dfrac{(n^2)!}{(n!)^n}$

But, these number of ways should be a natural number. Hence, $(n^2)!/(n!)^n$ is an integer.

Alternative Method 2:

We know that product of r consecutive integers is divisible by $r!$.

Now consider n^2 natural numbers.

These numbers can be divided in n rows each containing n consecutive integers as follows:

$1 \times 2 \times 3 \times \ldots \times n$

$\times (n + 1) \times (n + 2) \times (n + 3) \times \ldots \times (2n)$

$\times (2n + 1) \times (2n + 2) \times (2n + 3) \times \ldots \times (3n)$

$\times (3n + 1) \times (3n + 2) \times (3n + 3) \times \ldots \times (4n)$

...

...

...

$\times ((n - 1)n + 1) \times ((n - 1)n + 2) \times \ldots \times ((n - 1)n + n)$

Since product in each row is divisible by $n!$, the product of n such rows is divisible by $(n!)^n$.

Hence $(n^2)!/(n!)^n$ is an integer.